Name:_____ Date:_____

Math 10/11 Enriched: Section 4.8 Proving and Verifying Basic Trigonometric Identities

$\sin(-x) = -\sin x$ $\quad \tan x = \dfrac{\sin x}{\cos x}$ $\quad \csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x}$

$\cos(-x) = \cos x$ $\quad \sin^2 x + \cos^2 = 1$ $\quad \sin 2x = 2\sin x \cos x$

1. ***Verify*** which of the following are trigonometric identities:

a) $\tan x + \cot x = \sec x \csc x$	b) $\sec^2 x + \csc^2 x = \sec^2 x \bullet \csc^2 x$
c) $\sec^2 x - \csc^2 x = \dfrac{\sec^2 x}{\csc^2 x}$	d) $\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$
e) $\cos^2 x = \sin x(\csc x + \sin x)$	f) $\sin^2 x = \cos x(\sec x - \cos x)$
g) $\sin x \tan x + \sec x = \dfrac{\sin^2 x + 1}{\cos x}$	h) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$

2. Simplify each of the following expressions:

a) $\sin^2 x + \cos^2 x + \cot^2 x$	b) $\dfrac{\sin 2x}{1 + \cos 2x}$

c) $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$	d) $\cos(a+b)\cos b + \sin(a+b)\sin b$
e) $\sin\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)$	f) $\frac{\cos^3 x - \cos x}{\sin^3 x}$

3. Suppose $0 < x < 90°$ and $2\sin^2 x + \cos^2 x = \frac{25}{16}$. What is the value of $\sin x$?

4. Evaluate the following: $\sin\left(\frac{\pi}{6}\right)+\sin^2\left(\frac{\pi}{6}\right)+\cos^2\left(\frac{\pi}{6}\right)$

5. Determine the value of $\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{5\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right)$

6. Suppose that, for some angles "x" and "y" $\sin^2 x + \cos^2 y = \frac{3a}{2}$ and $\cos^2 x + \sin^2 y = \frac{1}{2}a^2$, determine the possible value(s) of "a".

7. What is the sum of all values of all values of "x" between 0 and 2π inclusive that satisfy the equation: $\tan x + 1 = \sec^2 x$?

8. If $\cos(x) = \frac{3}{4}$ and "x" is in the first quadrant, what is the value of $\sin(2x)$?

9. Suppose that $\sin a + \sin b = \sqrt{\frac{5}{3}}$ and $\cos a + \cos b = 1$. What is the value of $\cos(a-b)$?

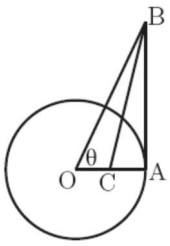
10. In degrees, what are all ordered pairs of angles (x,y) for which both angles are between 0 and $90°$ and satisfy the equation $\sin^2 x + \sin^2 y = \sin x + \sin y$?

11. In degrees, what are all values of 'x" between 0 *and* $360°$ for which $\sin x > \sqrt{1-\sin^2 x}$?

12. What are the degree measures of all positive angles between 0 and $90°$ which satisfy the equation: $\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x = 31$

13. A circle centered at "O" has radius 1 and contains the point A. Segment AB is tangent to the circle at "A" and $\angle AOB = \theta$. If point "C" lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then what is the length of OC?

a) $\sec^2\theta - \tan\theta$ b) $\frac{1}{2}$ c) $\frac{\cos^2\theta}{1+\sin\theta}$ d) $\frac{1}{1+\sin\theta}$ e) $\frac{\sin\theta}{\cos^2\theta}$



14. If $\cos\theta = 2\tan\theta$, solve for the numerical value of $\cos^2\theta$.

15. Simplify: $\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right)$

16. If $\sin x = \frac{-1}{3}$ and "x" is in quadrant 3, then what is the value of $\sin 2x$?

17. What is the value of $\sin(a+b)$ if $\sin a = \frac{-3}{5}$ and $\cos b = \frac{3}{5}$, with both "a" and "b" in the fourth quadrant.

18. Simplify the expression: $(\sin x - \cos x)^2 - (\sin x + \cos x)^2$

a) 0 b) $-\sin 2x$ c) $\sin 2x$ d) $-2\sin 2x$

19. Challenge: Evaluate and simplify the following without a calculator: $(\cos 36°)(\cos 108°)$

(a) Suppose that, for some angles x and y,

$$\begin{aligned} \sin^2 x + \cos^2 y &= \tfrac{3}{2}a \\ \cos^2 x + \sin^2 y &= \tfrac{1}{2}a^2 \end{aligned}$$

Determine the possible value(s) of a.